MATH 112

Exam 1 review

The following are review exercises for the Math 112 first exam. These exercises are provided for you to practice or test yourself for readiness for the first exam. You may use only a scientific calculator for the exam, but all answers must be EXACT: e.g., write $\frac{1}{3}, \frac{1}{2}$ (instead of 0.33, 0.5 respectively,) or $\sqrt{2}$ (instead of 1.41), π (instead of 3.14).

1. Perform the indicated operation and express your answer in simplest form with positive exponents only.
$$\frac{\left(x^{1/2}y^{1/3}\right)^{-2}\left(x^{1/3}y^{1/4}\right)^{-12}}{xy^{1/4}}$$

2. Factor completely.
   (a) $8n^2 - 2n - 3$ (b) $x^2 + 25 - 10x - 9y^2$ (c) $16x^4 - 81$ (d) $8x^3 - 1$
   (e) $3y(y-2)^2 + (y-2)$ (f) $2x(x^2+1)^{-1/3} + (x^2+1)^{2/3}$

3. Express the following as a simple fraction reduced to lowest terms.
   (a) $\dfrac{\dfrac{6}{x-4} + 2}{2 - \dfrac{4}{x-4}}$ (b) $\dfrac{\dfrac{4}{x^2} - \dfrac{3}{x}}{\dfrac{1}{x^2} + \dfrac{2}{3x}}$

4. Perform the indicated operations and simplify. Express your answers with positive exponents only.
   (a) $\dfrac{3y^{-1} + 4x^{-1}}{9y^{-2} - 16x^{-2}}$ (b) $(3 - a^{-2})^{-2}$ (c) $\dfrac{(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}}{(1-x^2)}$

5. Rationalize the denominator.
   (a) $\dfrac{2xy}{\sqrt[3]{4xy^2}}$ (b) $\dfrac{h}{\sqrt{x+h} + \sqrt{x}}$

6. Rationalize the numerator. $\dfrac{\sqrt{x+3} - \sqrt{5}}{x-2}$

7. Find the center and radius of the circle:

   (a) $(x-1)^2+y^2=8$ (b) $x^2+y^2+6x-4y=12$

8. Given $f(x)=3-x^2$ and $g(x)=\dfrac{2x}{x+3}$, find and simplify:

   (a) $\dfrac{f(x+h)-f(x)}{h}$ (b) $\dfrac{g(x+h)-g(x)}{h}$

9. Solve the equation explicitly for r: $s(r-3)=2-5(r+t)$

10. Find the exact domain and express your answer using interval notation.

    (a) $f(x)=\sqrt{\dfrac{x+2}{x-3}}$ (b) $f(x)=\sqrt{x+4}+\dfrac{1}{x^2}$ (c) $f(x)=\sqrt{49-x^2}$

    (d) $f(x)=\ln(x^2-x-20)$ (e) $f(x)=\log(4-x)+\sqrt{x-3}$ (f) $f(x)=\sqrt[3]{2x-5}+e^{x-7}$

11. Given $f(x)=x^2-1$, $g(x)=\ln(x+1)$ and $h(x)=e^x-2$, find and simply the following.

    (a) $(g\circ f)(x)$ (b) $(h\circ g)(x)$ (c) $(g\circ h)(0)$ (d) $(h\circ g)(0)$

12. Find the inverse of the following functions.

    (a) $f(x)=\dfrac{4x-1}{2x+3}$ (b) $f(x)=e^{2x-1}+3$ (c) $f(x)=\ln(x+3)-2$

13. Find the exact value of each expression.

    (a) $\log_8 2$ (b) $\ln\left(\dfrac{1}{e^3}\right)$ (c) $e^{\ln(\ln e^3)}$ (d) $\log_8 60-\frac{1}{2}\log_8 9-\log_8 5$

    (e) $e^{\ln 5-2\ln 7}$ (f) $\ln\sqrt{e}+e^{2\ln 3}$

14. Suppose $\log_5 x=A$ and $\log_5 y=B$. Rewrite $\log_5\left(\dfrac{25\sqrt[3]{x}}{y^2}\right)$ in terms of A and B only.

15. If $c=\log_{81}8$ and $d=\log_3 2$, what is the value of c in terms of d?

16. Solve the following equations for real solution.

    (a) $\dfrac{3}{4}x+\dfrac{5}{6}=2x-\dfrac{7}{3}$ (b) $(x-5)^2=12$ (c) $|2x+5|-3=7$

    (d) $\dfrac{x}{x+3}+\dfrac{5}{x-7}=\dfrac{30}{x^2-4x-21}$ (e) $\sqrt{7x+4}-2=x$ (f) $x^4-7x^2+12=0$

    (g) $\left(\dfrac{1}{3}\right)^{2-x}=27$ (h) $2^{5x+1}=e^2$ (i) $5^{2x-3}=7^{1-x}$ (j) $\log_5(x^2+2)=2$

    (k) $\log(x-15)+\log x=2$ (l) $\ln(x-4)-\ln x=\dfrac{1}{2}\ln 25$ (m) $e^{2x}+2e^x-3=0$

17. Solve the following inequality. Must show complete analysis where applicable. Express the solution using interval notation.

(a) $5a^2 \leq 7 - 34a$ (b) $\dfrac{3x+4}{2x+1} \leq 2$ (c) $4|3x-7| \geq 8$

18. For each problem below determine EXACTLY (if applicable) the vertex, $x-$ intercept(s), $y-$ intercept(s), vertical asymptoe(s), horizontal asymptote(s), domain, range, the interval(s) over which function is increasing/decreasing and end behavior. Then sketch the graph of the function.

(a) $f(x) = x^2 - x - 2$ (b) $f(x) = e^{x+1} - 3$ (c) $f(x) = -\ln(x+1) - 3$

(d) $f(x) = -3^{-x} + 2$ (e) $f(x) = \log_2(x-4) - 2$

19. Graph the piecewise function and find exact value of $f(0) - f(6)$.

$$f(x) = \begin{cases} 1 - x & if \quad x \leq -2 \\ x^2 & if \quad -2 < x \leq 2 \\ \log_2(x+2) & if \quad x > 2 \end{cases}$$

20. The weekly profit, $P(x) = -0.053x^2 + 15.9x + 7.5$ where x is the number of meals prepared. Find the number of meals that should be prepared to obtain the maximum profit. What is the maximum profit?

21. A farmer has 400 yards of fencing and wishes to fence three sides of a rectangular field (the fourth side is along an existing stone wall, and needs no additional fencing). Find the dimensions of the rectangular field of largest area that can be fenced.

22. Suppose \$5000 is invested at 7% interest compounded continuously. To the nearest year how long will it take for the investment to grow to \$15,000?

23. Jake invests \$30,000 in a bond that pays 6.5% interest compounded quarterly. How long will it take the investment to reach \$250,000. Round your answer to the nearest year.

24. The number of bacteria grows exponentially. A culture starts with 100 bacteria and triples every 5 hours. (a) Find the *exact* value of the growth constant r.(b) How many hours will it take for the bacteria to reach 900. (Express your answer as a positive integer).

25. The half life of cesium-137 is 30 days. Suppose you have 10 mg of sample.(a)Using the exponential decay model, write a formula for the mass remaining after t days, with r accurate to four decimal places.(b)To the nearest whole number, after how many days will only 2 mg of the sample remain?

Answers

1. $\dfrac{1}{x^6 y^{47/12}}$

2. (a) $(4n-3)(2n+1)$ (b) $(x-5-3y)(x-5+3y)$ (c) $(2x-3)(2x+3)(4x^2+9)$
   (d) $(2x-1)(4x^2+2x+1)$ (e) $(y-2)(3y^2-6y+1)$ (f) $\dfrac{(x+1)^2}{(x^2+1)^{1/3}}$

3. (a) $\dfrac{x-1}{x-6}$ (b) $\dfrac{12-9x}{3+2x}$

4. (a) $\dfrac{xy}{3x-4y}$ (b) $\dfrac{a^4}{9a^4-6a^2+1}$ (c) $\dfrac{1}{(1-x^2)^{3/2}}$

5. (a) $\sqrt[3]{2x^2y}$ (b) $\sqrt{x+h}-\sqrt{x}$

6. $\dfrac{1}{\sqrt{x+3}+\sqrt{5}}$

7. (a) Center: $(1,0)$, Radius: $2\sqrt{2}$ (b) Center: $(-3,2)$, Radius: 5

8. (a) $-2x-h$ (b) $\dfrac{6}{(x+3)(x+h+3)}$

9. $r=\dfrac{3s-5t+2}{s+5}$

10. (a) $(-\infty,-2]\cup(3,\infty)$ (b) $[-4,0)\cup(0,\infty)$ (c) $[-7,7]$
    (d) $(-\infty,-4)\cup(5,\infty)$ (e) $[3,4)$ (f) $(-\infty,\infty)$

11. (a) $2\ln x$ (b) $x-1$ (c) Undefined (d) -1

12. (a) $f^{-1}(x)=\dfrac{3x+1}{4-2x}$ (b) $f^{-1}(x)=\dfrac{\ln(x-3)+1}{2}$ (c) $f^{-1}(x)=e^{x+2}-3$

13. (a) $\dfrac{1}{3}$ (b) -3 (c) 3 (d) $\dfrac{2}{3}$ (e) $\dfrac{5}{49}$ (f) $\dfrac{19}{2}$

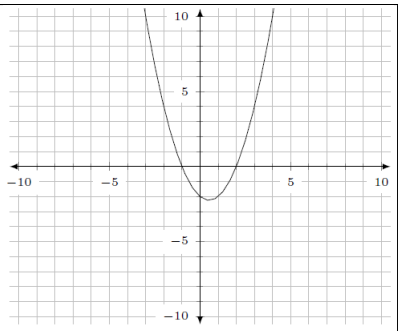
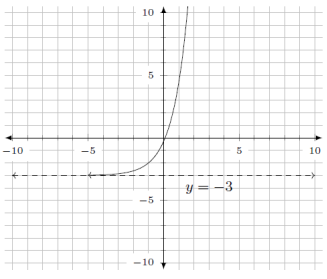
14. $2 + \dfrac{A}{3} - 2B$

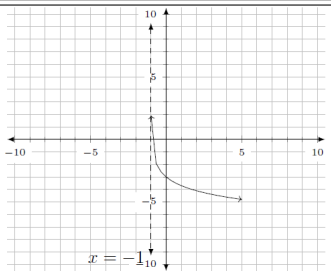
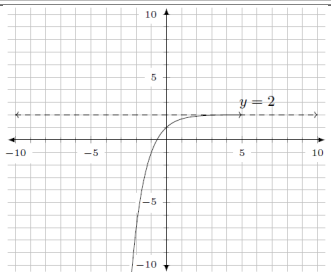
15. $c = \dfrac{3}{4}d$

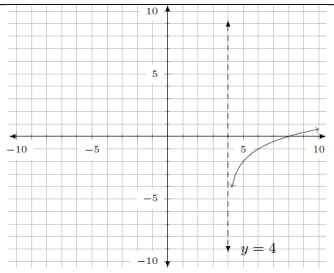
16. (a) $\dfrac{38}{15}$ (b) $x = 5 \pm 2\sqrt{3}$ (c) $x = -\dfrac{15}{2}, \dfrac{5}{2}$ (d) $x = 5$ (e) $x = 0, 3$
(f) $x = \pm 2, \pm\sqrt{3}$ (g) $x = 5$ (h) $x = \dfrac{2 - \ln 2}{5 \ln 2}$ (i) $x = \dfrac{\log 7 + 3 \log 5}{2 \log 5 + \log 7}$
(j) $x = \pm\sqrt{23}$ (k) $x = 20$ (l) ϕ or No solution (m) $x = 0$

17. (a) $[-7, \dfrac{1}{5}]$ (b) $(-\infty, -\dfrac{1}{2}) \cup [2, \infty)$ (c) $(-\infty, \dfrac{5}{3}] \cup [3, \infty)$

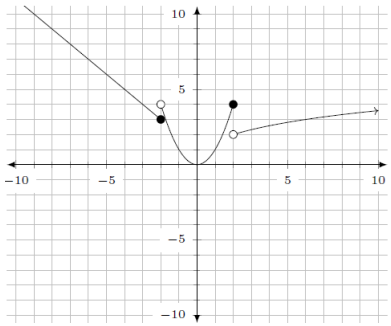
18.

	(a)	(b)
Vertex	$(\frac{1}{2}, -\frac{9}{4})$	NA
$x-$ intercept	$(2, 0), (-1, 0)$	$(\ln 3 - 1, 0)$
$y-$ intercept	$(0, -2)$	$(0, e - 3)$
Vertical asymptote(s)	NA	NA
Horizontal asymptote(s)	NA	$y = -3$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$
Range	$[-\frac{9}{4}, \infty)$	$(-3, \infty)$
Increasing	$[-\frac{9}{4}, \infty)$	$(-\infty, \infty)$
Decreasing	$(-\infty, -\frac{9}{4}]$	NA
End behavior	$y \to \infty$ as $x \to \infty$ $y \to \infty$ as $x \to -\infty$	$y \to \infty$ as $x \to \infty$ $y \to -3$ as $x \to -\infty$
Graph		

	(c)	(d)
Vertex	NA	NA
$x-$ intercept	$(e^{-3}-1, 0)$	$(-\frac{\ln 2}{\ln 3}, 0)$
$y-$ intercept	$(0, -3)$	$(0, 1)$
Vertical asymptote(s)	$x = -1$	NA
Horizontal asymptote(s)	NA	$y = 2$
Domain	$(-1, \infty)$	$(-\infty, \infty)$
Range	$(-\infty, \infty)$	$(-\infty, 2)$
Increasing	NA	$(-\infty, \infty)$
Decreasing	$(-1, \infty)$	NA
End behavior	$y \to -\infty$ as $x \to \infty$	$y \to 2$ as $x \to \infty$ $y \to -\infty$ as $x \to -\infty$
Graph		

	(e)
Vertex	NA
$x-$ intercept	$(8, 0)$
$y-$ intercept	NA
Vertical asymptote(s)	$x = 4$
Horizontal asymptote(s)	NA
Domain	$(4, \infty)$
Range	$(-\infty, \infty)$
Increasing	$(4, \infty)$
Decreasing	NA
End behavior	$y \to \infty$ as $x \to \infty$
Graph	

19.



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20. 150 meals and \$1200 21. 200 yds by 100 yds

22. 16 years 23. 33 years

24. (a) $r = \dfrac{\ln 3}{5}$ (b) 10 hours

25. (a) $m(t) = 10e^{-0.0231t}$ (b) 70 days