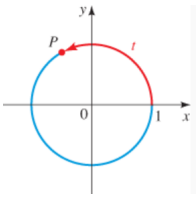
MATH 112

Exam II Review

The following are review exercises for the Math 112 second exam. These exercises are provided for you to practice or test yourself for readiness for the second exam. You may use only a scientific calculator for the exam, but all answers must be EXACT: e.g., write $\frac{1}{3}, \frac{1}{2}$ (instead of 0.33, 0.5 respectively,) or $\sqrt{2}$ (instead of 1.41), π (instead of 3.14).

1. The point $P(x, y)$ is on the unit circle in quadrant IV. If $x = \frac{\sqrt{11}}{6}$, find the *exact* value of y.

2. The point P in the figure below has $y-$ coordinate $\frac{4}{5}$, find the *exact* value(s) of the following.



   (a) $\sin t$ (b) $\cos t$ (c) $\tan t$ (d) $\sec t - \csc t$

3. For the given t, find (a) The *exact* value of the reference number determined by t and (b) the coordinates of the terminal point determined by t.

   (a) $t = \frac{7\pi}{4}$ (b) $t = -\frac{2\pi}{3}$ (c) $t = \frac{31\pi}{6}$ (d) $t = -\frac{54\pi}{3}$ (e) $t = -\frac{41\pi}{4}$

4. Find the *exact* value of the following.

   (a) $\cos\left(\frac{13\pi}{4}\right)$ (b) $\tan\left(-\frac{5\pi}{3}\right)$ (c) $\sin\left(-\frac{10\pi}{3}\right)$ (d) $\cot\left(-\frac{17\pi}{2}\right)$

5. Convert each degrees measure into radians for *exact* values.

   (a) $75°$ (b) $-120°$

6. Convert each radian measure into degrees for *exact* values.

   (a) $\frac{19\pi}{12}$ (b) $-\frac{3\pi}{4}$

7. Find one positive and one negative coterminal angle for each given angle.

   (a) $210°$ (b) $-650°$ (c) $-\frac{3\pi}{7}$ (d) $-\frac{13\pi}{3}$

8. Find an angle between 0 and 2π that is coterminal with the given angle.

   (a) $-\frac{5\pi}{3}$ (b) $\frac{17\pi}{4}$ (c) 10

9. Find an angle between 0 and 360° that is coterminal with the given angle.

   (a) $375°$ (b) $-800°$ (c) $1630°$

10. Given $f(x) = \sin x$ and $g(x) = \log_3(9 - x^2)$

    (a) Find the domain of $(f + g)(x)$ (b) Evaluate $(g \circ f)(\pi)$ for the *exact* value.

11. Given $x = \dfrac{\pi}{6}$, find the *exact* value of the following.

    (a) $\csc(2x)$ (b) $\sec^2(13x)$ (c) $\tan(5x)$

12. Find the length of an arc which subtends the central angle of 42° in a circle of diameter 60 inches. Round your answer to two decimal places.

13. An arc of length 87 cm subtends the circle of radius 16 cm. To the nearest tenth, find the degree measure of the central angle θ.

14. Find the area of the sector, to the nearest tenth given its central angle and radius.

    (a) $\theta = \dfrac{3\pi}{4}$, $r = 14$ cm (b) $\theta = 145°$, $r = 6$ ft

15. Express $\cot t$ in terms of $\cos t$, if the terminal point determined by t is in quadrant IV.

16. Given $\csc t = 5$, $\cos t < 0$, find the *exact* values of the other five trigonometric functions.

17. Given $\cos t = -\dfrac{2}{3}$ and if terminal point determined by t is in quadrant III, find the *exact* value of $\tan t \cos t + \csc t$

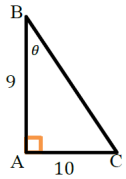
18. Determine whether the given functions are even, odd or neither.

    (a) $f(x) = \sin x + \cos x$ (b) $f(x) = x \sin^3 x$

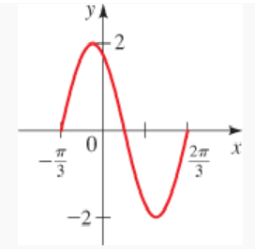
19. Given θ is an acute angle,find the *exact* values of other five trigonometric functions.

    (a) $\cos\theta = \dfrac{12}{13}$ (b) $\cot\theta = \dfrac{5}{3}$

20. Find the measure of the indicated angle.



21. The graph shown below is one period of a function of the form $y = a \sin k(x-b)$ or $y = a \cos k(x-b)$. Determine the amplitude, period, phase shift and function.



22. Given $t = 3.8$. (a) Plot approximate location of the terminal point on the unit circle.
(b) Determine the *exact* value of the reference number determined by t. (c) Find the coordinates of the terminal point determined by t, rounded to two decimal places.

23. Graph the functions below. Identify (when appropriate) the amplitude, period, phase shift, domain, range, $x-$intercepts and vertical asymptotes within that period.
(a) $f(x) = -2\sin\left(x + \frac{\pi}{4}\right)$ (b) $f(x) = 1 + \cos\left(2x - \frac{\pi}{3}\right)$ (c) $f(x) = -\tan\left(\frac{x}{3}\right) - 2$
(d) $f(x) = \frac{1}{2}\cot\left(\frac{2}{3}x - \pi\right)$ (e) $f(x) = 2 - \sec\left(x + \frac{5\pi}{6}\right)$

24. Standing 50 feet from the base of a tree, a person looks up to the top with an angle of elevation of $82°$. To the nearest foot, find the height of the tree.

25. The distance from a helicopter to a tower is 400 feet and the angle of depression is $37°$. To the nearest foot, find the distance, on the ground from a point directly below the helicopter to the tower.

26. From the top of a building, a man observes a car moving toward him. As the car moves 200 ft closer, the angle of depression changes from $25°$ to $45°$. To the nearest foot, find the height of the building.

27. A pilot measure the angles of depression to two ships to be $40°$ and $52°$. If the pilot is flying at an elevation of 35,000 ft, find the distance between two ships. Round your answer to the nearest foot.

28. In a predator/prey model, the predator population is modeled by the function $y(t) = 650\cos\left(\frac{\pi}{3}t\right) + 5,000$, where t is measured in months, and $t = 1$ is January. (a) What is the maximum population (b) What is the minimum population? (c) Find the length of time between successive periods of maximum population.(d) In which calendar month is the population lowest?

Answers

1. $y = -\dfrac{5}{6}$
2. (a) $\dfrac{4}{5}$ (b) $-\dfrac{3}{5}$ (c) $-\dfrac{4}{3}$ (d) $-\dfrac{35}{12}$
3. (a) Reference number: $\dfrac{\pi}{4}$, Terminal point $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$
   (b) Reference number: $\dfrac{\pi}{3}$, Terminal point $\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$
   (c) Reference number: $\dfrac{\pi}{6}$, Terminal point $\left(-\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$
   (d) Reference number: 0, Terminal point $(1, 0)$
   (e) Reference number: $\dfrac{\pi}{4}$, Terminal point $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$
4. (a) $-\dfrac{\sqrt{2}}{2}$ (b) $\sqrt{3}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) 0
5. (a) $\dfrac{5\pi}{12}$ (b) $-\dfrac{2\pi}{3}$
6. (a) $285°$ (b) $-135°$
7. (a) $570°, -150°$ (b) $70°, -290°$ (c) $\dfrac{11\pi}{7}, -\dfrac{17\pi}{7}$ (d) $\dfrac{5\pi}{3}, -\dfrac{\pi}{3}$
8. (a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{4}$ (c) 3.719 radians
9. (a) $15°$ (b) $280°$ (c) $190°$
10. (a) $(-3, 3)$ (b) 2
11. (a) $\dfrac{2\sqrt{3}}{3}$ (b) $\dfrac{4}{3}$ (c) $-\dfrac{\sqrt{3}}{3}$
12. 21.99 inches
13. 311.5^o
14. (a) 230.9 cm^2 (b) 45.6 ft^2
15. $-\dfrac{\cos t}{\sqrt{1 - \cos^2 t}}$
16. $\sin t = \dfrac{1}{5}$ $\cos t = -\dfrac{2\sqrt{6}}{5}$ $\sec t = -\dfrac{5\sqrt{6}}{12}$ $\tan t = -\dfrac{\sqrt{6}}{12}$ $\cot t = -2\sqrt{6}$
17. $-\dfrac{14\sqrt{5}}{15}$

18. (a) Neither (b) Even

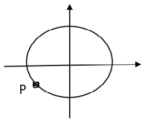
19. (a) $\sin\theta = \dfrac{5}{13}$ $\tan\theta = \dfrac{5}{12}$ $\sec\theta = \dfrac{13}{12}$ $\csc\theta = \dfrac{13}{5}$ $\cot\theta = \dfrac{12}{5}$

(b) $\sin\theta = \dfrac{3\sqrt{34}}{34}$ $\cos\theta = \dfrac{5\sqrt{34}}{34}$ $\tan\theta = \dfrac{3}{5}$ $\sec\theta = \dfrac{\sqrt{34}}{5}$ $\csc\theta = \dfrac{\sqrt{34}}{3}$

20. $\tan\theta = \dfrac{10}{9}$

21. Amplitude: 2 Period: π Phase shift: $-\dfrac{\pi}{3}$ Function: $y = 2\sin 2\left(x + \dfrac{\pi}{3}\right)$

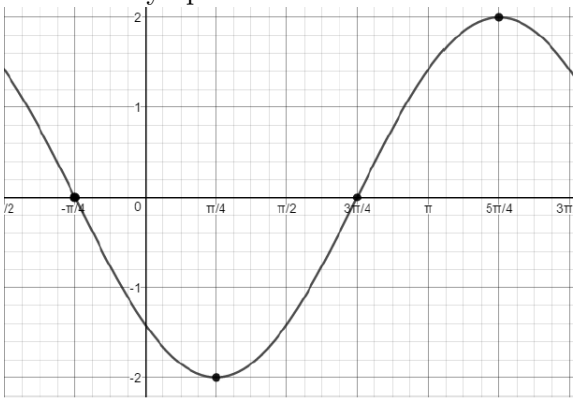
22. (a)



(b) $3.8 - \pi$ (c) $(-0.79, -0.61)$

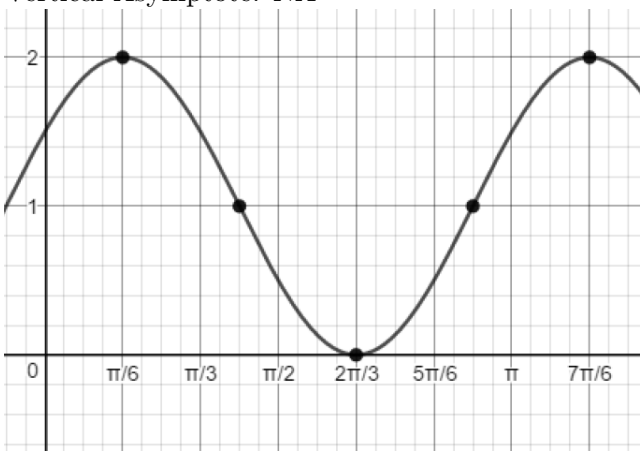
23. (a) Amplitude: 2 Period: 2π Phase Shift: $-\dfrac{\pi}{4}$ Domain: All Reals, Range: $[-2, 2]$

Vertical Asymptote: NA

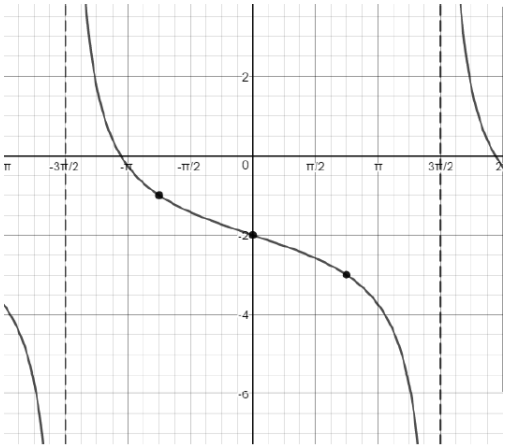


(b) Amplitude: 1 Period: π Phase Shift:$\frac{\pi}{6}$ Domain: All Reals, Range: $[0, 2]$

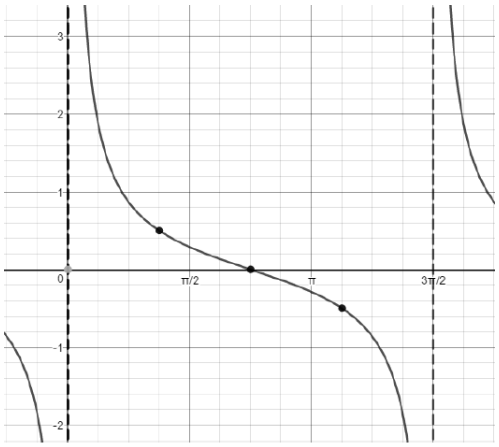
Vertical Asymptote: NA



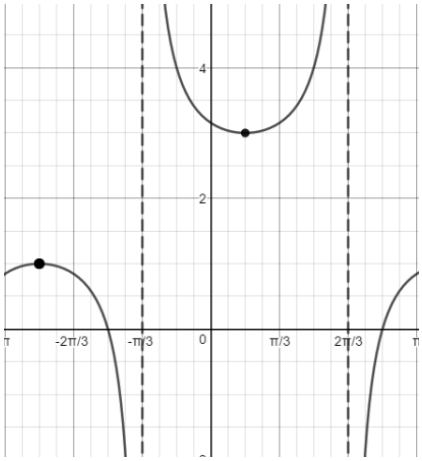
(c) Amplitude: NA Period: 3π Phase Shift: None Domain: $x \neq -\frac{3\pi}{2}, \frac{3\pi}{2},$ Range: All Reals Vertical Asymptote: $x = -\frac{3\pi}{2}, \frac{3\pi}{2}$



(d) Amplitude: NA Period: $\frac{3\pi}{2}$ Phase Shift: $\frac{3\pi}{2}$ Domain: $x \neq 0, \frac{3\pi}{2},$ Range: All Reals Vertical Asymptote: $x = 0, \frac{3\pi}{2}$



(e) Amplitude: NA Period: 2π Phase Shift: $-\frac{\pi}{3}$ Domain: $x \neq -\frac{\pi}{3}, \frac{2\pi}{3}$,
Range: $(-\infty, 1] \cup [3, \infty)$ Vertical Asymptote: $x = -\frac{\pi}{3}, \frac{2\pi}{3}$



24. 356 feet

25. 319 feet

26. 175 feet

27. 14,366 feet

28. (a) 5650 (b) 4350 (c) 6 (d) March