MATH 112
Final Exam Review

The following are review exercises for the Math 112 final exam. These exercises are provided for you to practice or test yourself for readiness for the final exam. There are many more problems appearing here than would be on the final. These exercises represent many of the types of problems you would be expected to solve on the final, but are not meant to represent all possible types of problems that could appear on the final exam. To best prepare, students may also want to review their recitation quizzes, both of the midterms, and redo Math 112 exams 1 & 2 review exercises posted on the course homepage. Your final exam will be one part. You may use only a scientific calculator for the exam, but all answers must be exact. Unless otherwise stated, give EXACT answers: e.g., write $\frac{1}{3}, \frac{1}{2}$ (instead of $0.33, 0.5$ respectively), $\sqrt{2}$ (instead of 1.41), π (instead of 3.14) or $\log(2)$ (instead of 0.3010).

1. Factor completely:

   (a) $2x^{7/2} - 5x^{5/2} - 3x^{3/2}$ (b) $(x^2+1)^{1/2} - 5(x^2+1)^{-1/2}$

2. Solve the following for real values of x:

   (a) $x^4 + 3x^2 - 4 = 0$ (b) $|3x - 2| + 4 = 5$ (c) $|5x + 2| + 1 \leq 5$

3. Find the equation of a circle in standard form with center $(-1, 3)$ and radius 4.

4. Given $f(x) = 2x^2 - 3x + 1$ and $g(x) = \dfrac{x}{x-1}$, find and simplify the following:

   (a) $\dfrac{f(x+h) - f(x)}{h}, h \neq 0$ (b) $\dfrac{g(x+h) - g(x)}{h}, h \neq 0$

5. Sketch the graph of $f(x) = \begin{cases} 1+x & if \quad x < -1 \\ x^2 & if \quad -1 \leq x < 1 \\ 2-x & if \quad x \geq 1 \end{cases}$

6. Find the domain of the following functions and express your answer using interval notation.

   (a) $f(x) = \dfrac{x+4}{x^2-9}$ (b) $f(x) = \sqrt{x^2 - 7x - 18}$ (c) $f(x) = e^{x-3} + 2$

   (d) $f(x) = \sqrt{3-x} + \log(x+4)$ (e) $f(x) = \ln(x^2 - 16)$

7. Given $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{x}{x+3}$, find and simplify $(g \circ f)(x)$

8. Find the Maximum/Minimum value of the function $f(x) = 2x^2 + 7x + 5$.

9. Sketch the graph of the following: Using interval notation, state the domain and the range. State the equation(s) of the asymptote (s). Find the value(s) of $x-$ and $y-$ intercepts where they exist.

   (a) $f(x) = 3^{x-2} + 4$ (b) $f(x) = \ln(x-2) + 3$

   (c) $f(x) = e^{x+3} - 1$ (d) $f(x) = \log(x+1) - 2$

10. Given $\log_b 5 = 2.3$ and $\log_b 3 = 1.6$, fill in the table below with the appropriate values.

x	15	9	$\frac{5}{3}$	$5a$	$\frac{3}{a^2}$
$\log_a x$					

11. Which is larger: $\log_3 28$ or $\log_4 63$? Justify your answer for full credit.
12. Solve the following:

    (a) $\log(x+2) - \log(x) = 3$ (b) $\log(x-3) = 1 - \log(x)$ (c) $2^{2x+1} = 8^{3x}$

    (d) $3^{2x-3} = 5^{x+3}$ (e) $x^2e^x - 2e^x = 0$

13. Given the following equations: Find (i) solutions in the interval $[0, 2\pi)$ in radians, and (ii) all solutions in radians:

    (a) $(2\sin x - \sqrt{3})(\sin x - 2) = 0$ (b) $2\sin^2 x + \sin x - 3 = 0$ (c) $\tan^2 x - 1 = 0$

14. For the functions below, find the period, amplitude, phase shift, and sketch the graph of one period. Be sure to label all $x-$ intercepts and maximum and minimum points within one period.

    (a) $f(x) = 6\cos\left(2x - \dfrac{\pi}{3}\right)$ (b) $f(x) = -5\sin\left(3x + \dfrac{\pi}{2}\right)$

15. Find the value of the following:

    (a) $\sin(15°)$ (b) $\cos(75°)$

16. Find the reference angle for $-135°$. Find value of $\cos(-135°)$
17. (a) If $\cos t = \dfrac{4}{7}$ and $\dfrac{3\pi}{2} < t < 2\pi$, find the remaining values of the trigonometic functions of t.

    (b) If $\sin t = -\dfrac{3}{8}$ and $\pi < t < \dfrac{3\pi}{2}$, find the remaining values of the trigonometic functions of t.
18. Find the values of the trigonometric functions of θ if the terminal side of θ contains the point $(-4, 5)$
19. Find:

    (a) Given $\cos x = -\dfrac{4}{5}$ and x is in quadrant II, find $\cos(2x)$

    (b) Given $\sin x = \dfrac{2}{5}$ and x is in quadrant II, find $\sin(2x)$

    (c) Given $\sin x = -\dfrac{2}{3}$ and x is in quadrant III,and $\sin y = \dfrac{1}{3}$ and y is in quadrant II, find $\sin(x+y)$
20. Verify the identity:

    (a) $\dfrac{1 - \sin^2 x}{\cot^2 x} = \sin^2 x$ (b) $\dfrac{\cos x}{1 + \sin x} + \dfrac{\cos x}{1 - \sin x} = 2\sec x$

21. Evaluate the following in degrees:

    (a) $\sin^{-1}\left(-\dfrac{1}{2}\right)$ (b) $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

22. Evaluate the following:

    (a) $\sin\left[\cos^{-1}\left(\dfrac{2}{3}\right)\right]$ (b) $\sin\left[\tan^{-1}\left(-6\right)\right]$

23. If $\theta = \arcsin x$, express $\tan\theta$ in terms of x.

24. (a) Suppose on Jan 1, 1997 Dave invested \$2,000 into a bank account at 5% interest compounded continuously. Let $y(t)$ be the value of Dave's investment after t years. Give an exact formula for $y(t)$.

    (b) Also on Jan 1, 1997 John decides to invest. He put \$2,500 into an account at 3% interest compounded monthly. Let $g(t)$ be the value of John's investment after t years. Give an exact formula for $g(t)$.

    (c) Which account is worth more after 9 years? [Must show work to receive credit.]

    (d) To the nearest tenth, at what time t is the value of both accounts the same?

25. Evaluate the following:

    (a) $\log_2 \sqrt[7]{64}$ (b) $\log_3 \sqrt[7]{81}$

26. Evaluate the following. Round your answer to two decimal places.

    (a) $\log_5 27$ (b) $\log_3 15$

27. Given the following functions $f(x)$ below, find $f^{-1}(x)$.

    (a) $f(x) = 3^x$ (b) $f(x) = \log_3 x$

    (c) $f(x) = 10^{x+1} - 3$ (d) $f(x) = \log(x-4) + 5$

    (e) $f(x) = e^{2x+1} - 4$ (f) $f(x) = \ln(2x+3) - 5$

28. Given $f(x) = e^{x-3} + 2$ and $g(x) = \ln(x-2) + 1$, find the following:

    (a) $f^{-1}(4)$ (b) $g^{-1}(5)$ (c) $(g \circ f)(x)$

29. From 1990 to 2000 the student tuition at a University grew from \$12,000 to \$18,000.

    (a) Using the exponential growth model, determine r, the annual rate of increase for the population as a decimal accurate to 3 places.

    (b) Assuming the same growth rate use r found in Part (a) above, find in what year (to the nearest year) the tuition of the University will reach \$30,000.

30. Carbon dating is commonly used to determine how old an object is by measuring the amount of carbon-14 that is left in an object as the object decays over the years. This decay proceeds exponentially with a half-life of approximately 5800 years. How old (to the nearest year) would carbon dating say a piece of bone is when the amount of carbon-14 has decayed from its original amount of 100 grams to a final amount of 22 grams?

31. Air Temperature T varies in a periodic manner, with highs during the day and lows during the night. Assume that the weather pattern is the same for one week, and suppose that T (in degrees Fahrenheit) at a particular time of day is given by the function.

    $T(t) = 13 \sin\left(\frac{\pi}{12}t - \frac{2\pi}{3}\right) + 62$, where t is the time in hours (with $t = 0$ at midnight).

    (a) What is the highest temperature during the day?

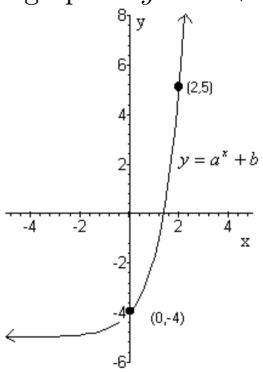
    (b) What is the lowest temperature during the day?

    (c) Find the temperature at 12 PM.

    (d) Find the temperature at 6 PM.

    (e) What is the period of $T(t)$?

32. The graph of $y = a^x + b$ is shown below. Find the values of a and b.



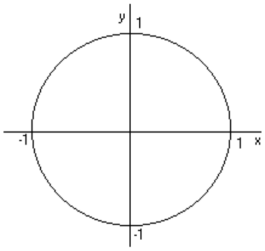
33. A ramp 17 feet in length rises to a loading platform that is 4 feet off the ground. Find the measure of the angle θ in degrees that the ramp makes with the ground. (Give your answer accurate to 2 decimal places.)

34. Find the length of a $25°$ arc in a circle of radius 12 inches, accurate to 2 decimal places.

35. Two trains, Train A and Train B, leave a train station at 10:00 AM traveling along straight tracks at 80 and 90 mi/hr, respectively. If the angle between their directions of travel is $118°$, to the nearest mile, how far are the trains from each other at 11:30 AM?

36. Given $t = 4$, complete the following:

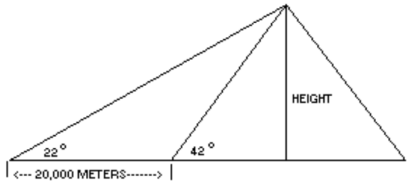
    (a) Using $t = 4$, sketch on the unit circle the approximate location of $P(x, y)$, the terminal point.



    (b) Find the reference number for t (to two decimal places)

    (c) Find the coordinates of the terminal point determined by t ?(Give to two decimal places)

37. The top of a volcano is viewed from a safe distance of 20,000 meters level to the base of the volcano. The angle of elevation is found to be 22 degrees. If the angle of incline from the base of the volcano to its summit is found to be 42 degrees, to the nearest foot, how high is the volcano?



38. A pilot in an airplane flying at 25,000 ft sees two towns directly ahead of her in a straight line. The angles of the depression to the towns are $25°$ and $50°$, respectively. To the nearest foot, how far apart are the towns?

ANSWERS: MATH 112 FINAL EXAM REVIEW EXERCISES

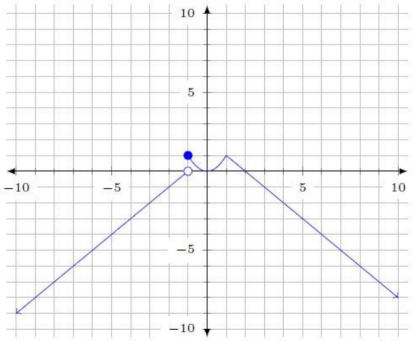
1. (a) $x^{3/2}(2x+1)(x-3)$ (b) $(x^2+1)^{-1/2}(x-2)(x+2)$

2. (a) $x=-1,1$ (b) $x=\frac{1}{3},1$ (c) $-\frac{6}{5}\leq x\leq\frac{2}{5}$

3. $(x+1)^2+(y-3)^2=16$

4. (a) $4x+2h-3$ (b) $\dfrac{-1}{(x-1)(x+h-1)}$

5. Graph of $f(x)$

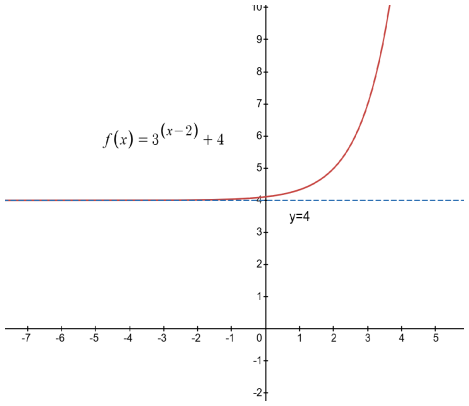


6. (a) $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ (b) $(-\infty,-2]\cup[9,\infty)$ (c) $(-\infty,\infty)$
(d) $(-4,3]$ (e) $(-\infty,-4)\cup(4,\infty)$

7. $\dfrac{1}{1+3x}$

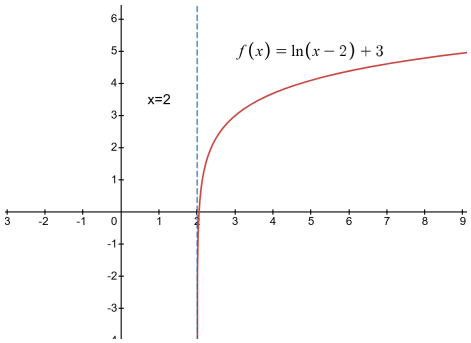
8. Minimum value is $-\dfrac{9}{8}$

9. (a)



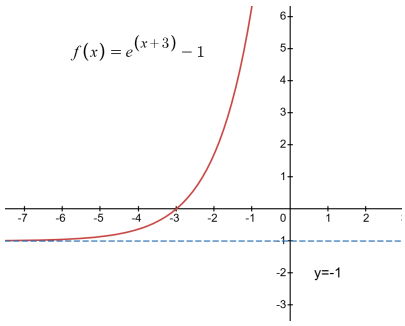
Domain: $(-\infty,\infty)$
Range: $(4,\infty)$
Asymptote: $y=4$
$x-$ intercept: None
$y-$ intercept: $\dfrac{37}{9}$

(b)



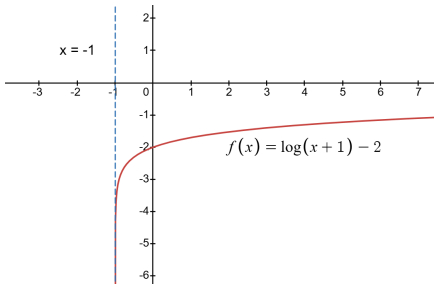
Domain: $(2,\infty)$
Range: $(-\infty,\infty)$
Asymptote: $x=2$
$x-$ intercept: $(e^{-3}+2,0)$
$y-$ intercept: None

(c)



Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$
Asymptote: $y = -1$
$x-$ intercept: $(-3, 0)$
$y-$ intercept: $(0, e^3 - 1)$

(d)



Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$
Asymptote: $x = -1$
$x-$ intercept: $(99, 0)$
$y-$ intercept: $(0, -2)$

10.

x	15	9	$\frac{5}{3}$	$5a$	$\frac{3}{a^2}$
$\log_a x$	3.9	3.2	0.7	3.3	-0.4

11. If $\log_3 28 = x \implies 3^x = 28$. Hence $x > 3$. On the other hand, $\log_4 63 = y \implies 4^x = 63$. Hence $y < 3$.Therefore $\log_3 28 > \log_4 63$.

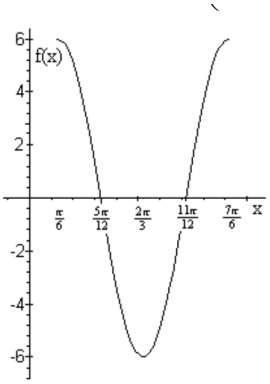
12.
(a) $x = \dfrac{2}{999}$
(b) $x = 5$
(c) $x = \dfrac{1}{7}$
(d) $x = \dfrac{3\ln 5 + 3\ln 3}{2\ln 3 - \ln 5}$
(e) $x = \pm\sqrt{2}$

13. (a) (i) $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ (ii) $x = \dfrac{\pi}{3} + 2n\pi, \dfrac{2\pi}{3} + 2n\pi$, n is an interger.

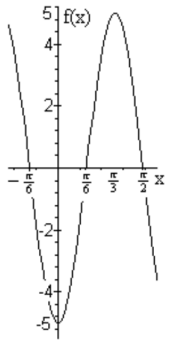
(b) (i) $x = \dfrac{\pi}{2}$ (ii) $x = \dfrac{\pi}{2} + 2n\pi$, n is an interger.

(c) (i) $x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ (ii) $x = \dfrac{\pi}{4} + 2n\pi, \dfrac{3\pi}{4} + 2n\pi, \dfrac{5\pi}{4} + 2n\pi, \dfrac{7\pi}{4} + 2n\pi$, n is an interger.

14. (a)



Period: π
Amplitude: 6
Phase shift: $\dfrac{\pi}{6}$ right

(b) 

Period: $\dfrac{2\pi}{3}$
Amplitude: 5
Phase shift: $\dfrac{\pi}{6}$ left

15. (a) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

16. $45°$, $-\dfrac{\sqrt{2}}{2}$

17. (a) $\sin t = -\dfrac{\sqrt{33}}{7}$, $\tan t = -\dfrac{\sqrt{33}}{4}$, $\cot t = -\dfrac{4}{\sqrt{33}}$, $\sec t = \dfrac{7}{4}$, $\csc t = -\dfrac{7}{\sqrt{33}}$

(b) $\cos t = -\dfrac{\sqrt{55}}{8}$, $\tan t = \dfrac{3}{\sqrt{55}}$, $\cot t = \dfrac{\sqrt{55}}{3}$, $\sec t = -\dfrac{8}{\sqrt{55}}$, $\csc t = -\dfrac{8}{3}$

18. $\sin t = \dfrac{5}{\sqrt{41}}$, $\cos t = -\dfrac{4}{\sqrt{41}}$, $\tan t = -\dfrac{5}{4}$, $\cot t = -\dfrac{4}{5}$, $\sec t = -\dfrac{\sqrt{41}}{4}$, $\csc t = \dfrac{\sqrt{41}}{5}$

19. (a) $\dfrac{7}{25}$ (b) $-\dfrac{4\sqrt{21}}{25}$ (c) $\dfrac{4\sqrt{2}-\sqrt{5}}{9}$

20. (a) $\dfrac{1-\sin^2 x}{\cot^2 x} = \dfrac{\cos^2 x}{\cot^2 x} = \dfrac{\cos^2 x}{\dfrac{\cos^2 x}{\sin^2 x}} = \cos^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x} = \sin^2 x$

(b)
$$\begin{aligned}
\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} &= \frac{(1-\sin x)\cos x}{(1-\sin x)(1+\sin x)} + \frac{(1+\sin x)\cos x}{(1-\sin x)(1+\sin x)} \\
&= \frac{(1-\sin x)\cos x + (1+\sin x)\cos x}{(1-\sin x)(1+\sin x)} \\
&= \frac{\cos x - \sin x\cos x + \cos x + \sin x\cos x}{(1-\sin x)(1+\sin x)} \\
&= \frac{2\cos x}{(1-\sin x)(1+\sin x)} = \frac{2\cos x}{1-\sin^2 x} \\
&= \frac{2\cos x}{\cos^2 x} = \frac{2}{\cos x} = 2\sec x
\end{aligned}$$

21. (a) $-30°$ (b) $150°$

22. (a) $\dfrac{\sqrt{5}}{3}$ (b) $-\dfrac{6}{\sqrt{37}}$

23. $\dfrac{x}{\sqrt{1-x^2}}$

24. (a) $y(t) = 2000e^{0.05t}$ (b) $g(t) = 2500\left(1+\dfrac{0.03}{12}\right)^{12t}$

(c) $y(9) = \$3,136.62$ and $g(9) = \$3,273.81$. Hence John's account was worth more.

(d) $t = 11.1$ years

25. (a) $\frac{6}{7}$ (b) $\frac{4}{7}$

26. (a) 2.05 (b) 2.46

27. (a) $f^{-1}(x) = \log_3 x$ (b) $f^{-1}(x) = 3^x$ (c) $f^{-1}(x) = \log(x+3) - 1$
(d) $f^{-1}(x) = 10^{x-5} + 4$ (e) $f^{-1}(x) = \dfrac{\ln(x+4) - 1}{2}$ (f) $f^{-1}(x) = \dfrac{e^{x+5} - 3}{2}$

28. (a) $f^{-1}(4) = \ln(2) + 3$ (b) $g^{-1}(5) = e^4 + 2$ (c) $(g \circ f)(x) = x - 2$

29. (a) $r \approx 0.041$ (b) 2012

30. $12,670$ years old

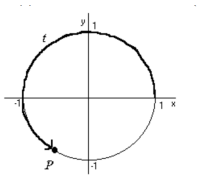
31. (a) $75°$ (b) $49°$ (c) $62+\dfrac{13\sqrt{3}}{2}$ degrees (d) $68.5°$
(e) 24 hours

32. (a) $a = \sqrt{10}$ (b) $b = -5$

33. $13.61°$

34. 5.24 inches

35. 219 miles

36. (a) 

(b) 0.86 (c) $P(-0.65, -0.76)$

37. $14,658$ meters

38. $32,635$ feet