Mathematics 552 **Algebra II** **Spring 2017**

Week 4 Completely Reducible modules and Semisimple rings
Jacobson II: 4.1 - 4.4

1. Jacobson II 4.1.4
2. Jacobson II 4.1.5
3. Jacobson II 4.2.2
4. Jacobson II 4.2.6
5. Jacobson II 4.3.7 (try to show the implications in the order iii), ii), i), iii) using the Krull-Schmidt theorem and the definition of completely reducible)
6. Jacobson II 4.4.2 (various authors differ on the definition of simple ring, but all agree that a two sided ideal in a simple ring is 0 or the whole ring, so use that property in this and the next problem.)
7. Jacobson II 4.4.3. You can answer this question without using any structure theorems. Just use the definitions and the previous problem to analyze the center.
8. Let R be a ring.
   a) Show that R is semisimple if and only if every left R-module is projective.
   b) Show that R is a division algebra if and only if every left R-module is free.